INFLUENCES OF LOADING PULSE WIDTH AND LOADING AREA SIZE ON WAVE PROPAGATION IN RODS

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Abstract

Based on the dispersion of longitudinal modes and the three-dimensional finite element analysis for wave propagation in cylindrical rods, a research was carried out to study the influence of the impact pulse width on the velocity responses of particles at different positions of rods, and also to investigate the velocity responses of the particles at the top of rods affected by the ratio of the radius of loading area to the radius of rods. Influenced by the dispersion of longitudinal modes of waves and the ratio of the radius of loading area to the radius of rods. Influenced by the dispersion in cylindrical rods. One-dimensional wave theory is not available to analyze the wave propagation in cylindrical rods. One-dimensional wave theory can only be applied approximately to analysis of waves propagating in rods if the ratio of the impulse width to the diameter of rods is greater than 4, and the ratio of the radius of loading area to the radius of rods is greater than 1/4 and the waves are reflected from positions which are more than 4 times the diameter below the top of rods. This article also revealed that during the low strain integrity testing of piles, the wave reflections from the pile toes and their consecutive waves are helpful to differentiate the reflections from the shallow part of piles.

Keywords: dispersion, impulse width, impact pulse width, one-dimensional approximation, wave impedance, integrity testing of piles

1. Introduction

The impact-echo technique has widely been applied to test the integrity of piles. The signals measured on the top of piles are used to analyze qualitatively or quantitatively the location, extent, and degree of impedance variation of piles, based on one dimensional stress wave theory (1-D wave theory). However, propagation of waves near the top of piles is actually in three dimensions. When the impedance of pile varies, the reflected waves in different particles on the top will have different velocity responses. Some particles on the top will have great response to the reflected waves, some may have little response and may even be difficult to distinguish from the signals measured on the top.

In practical integrity testing of piles, in order to analyze the signals conveniently, some assumptions are made:

- the first, the waves propagating down a distance from the top are regarded as non-dispersively plane waves;
- the second, the first pulse of the signal measured on the top is often regarded as the particle velocity pulse of the plane waves.

The assumptions are mainly affected by the ratio of the radius of loading area to the radius of piles and the dispersion of waves which is closely related to the ratio of the width of the impact force impulse to the diameter of piles (in this paper, the pulse width is defined as the product of half a sinusoidal force

impulse duration and the wave velocity of C0). Therefore, the investigation on the influences of dispersion of waves and the loading area on one-dimensional approximation are helpful for improving the accuracy of the analysis of integrity of piles.

Tang^[4] discussed the influences of two-dimensional waves on the particle velocity responses by the finite element method and concluded that if the measured signals are analyzed approximately by 1-D wave theory, the available positions for the transducer are within 1/2R~3/4R from the axis of piles. Chen^[3] investigated the oscillation with high frequency signals measured on the top by analyzing the reflection of shear wave and Rayleigh wave and obtained the positions with small oscillation; and he also analyzed the influences of impact impulse width on 1-D approximation by numerical simulation. Their numerical results explained the phenomena but they did not analyze the problems based on theory.

Liao^{[5][6]} et. al. discussed the possibility of determining the pile length and the location of the impedance variation by the mechanical admittance technique using finite element analysis results. However, he did not pay attention to the problem of one-dimensional approximation.

This article attempts to analyze the responses of the particles from the dispersion of the waves propagating in cylindrical rods and the influences of the higher mode waves, and to discuss the necessary condition for one-dimensional approximation as well as the way to improve the accuracy of the integrity testing of piles.

In general, the shape of cross-section of piles is approximated to circle. The studies of this article will focus on the waves propagating in cylindrical rods. Due to the interaction between pile and soil, theoretical analysis of the dispersions become difficult, and its numerical analysis results will be presented in other article.

2. Dispersion of the waves propagating in cylindrical rods

2.1 Dispersion of longitudinal modes

In the classical one-dimensional wave propagation theory, waves with different frequencies propagate at the same velocities. According to Fourier analysis, transient waves can be expanded into a series of harmonic waves with different frequencies and amplitudes. Due to harmonic waves with different frequencies propagating at the same velocity, the pattern of transient waves does not change during the wave propagation. However, waves with different frequencies propagate at different wave velocities in rods, hence dispersion of waves occurs.

For a symmetrical rod, the frequency equation for longitudinal wave can be derived from equilibrium equation, geometric equation, physical equation and free boundary conditions^[1]:

$$\frac{2\alpha}{R} \left(\beta^2 + \xi^2\right) J_1(\alpha \cdot R) J_1(\beta \cdot R) - \left(\beta^2 - \xi^2\right)^2 J_0(\alpha \cdot R) J_1(\beta \cdot R) - 4\xi^2 \alpha \beta J_1(\alpha \cdot R) J_0(\beta \cdot R) = 0 \quad (1)$$

where R is radius of the rod,

$$\alpha^{2} = \frac{\omega^{2}}{C_{1}^{2}} - \xi^{2}, \beta^{2} = \frac{\omega^{2}}{C_{2}^{2}} - \xi^{2}, C_{1} = \sqrt{\frac{(1-2\nu)}{(1+\nu)(1-\nu)}} \frac{E}{\rho}, C_{2} = \sqrt{\frac{1}{2(1+\nu)}} \frac{E}{\rho}, \omega = \frac{2\pi}{f}, \xi = \frac{2\pi}{\lambda}, \varepsilon$$

f, λ , ν are frequency, wave length and Poisson's ratio, J₀, J₁ are Bessel functions of the zeroth and the first orders respectively. Equation (1) can be solved by numerical method. The dispersion curves for the first three longitudinal modes for Poisson's ratio of 0.29 are shown in Fig. 1.

where
$$\overline{C} = C/C_0$$
, $\overline{\xi} = \frac{R \cdot \xi}{2\pi} = \frac{R}{\lambda}$, $C_0 = \sqrt{\frac{E}{\rho}}$

From the Figure 1, when $\overline{\xi} = R/\lambda$ becomes very small, i.e. λ becomes very large, only the fundamental mode exists, and the phase velocity of longitudinal wave approaches the classical bar velocity. However, the deviation from C₀ increases with increasing of $\overline{\xi}$. As $\overline{\xi}$ increases to large values, the phase velocity approaches to that of Rayleigh wave, the disturbance induced by the longitudinal waves is mainly confined to the surface.

When $\overline{\xi}$ exceeds the cut off $\overline{\xi}_0$ of higher modes, influences of higher modes on wave propagation take effect, the number of higher modes and their influences increase with increasing of $\overline{\xi}$. As shown in Fig. 1, when $\overline{\xi} < 0.1$, i.e. $\lambda > 10$ R, the fundamental longitudinal wave mode propagates at the velocity close to that of the classical rod.





2.2 Analysis on the propagation of longitudinal waves

The theoretical solution to the stress wave induced by impact and propagating in cylindrical rods is quite difficult. If only the effects of lateral inertial on propagation of waves are considered, the solution will be simplified. The relation between the phase velocity and wave-number can be obtained from Love's equation:

$$C/C_{o} = \frac{1}{(1+(k\nu\gamma)^{2})^{1/2}}$$
 (2)

where $k = \sqrt{J/A}$, J is polar moment of inertia, A is cross-sectional area, $k = R/\sqrt{2}$ for circular

cross-section.

Davies utilized Love's equation in analyzing the propagation of sharp pulses in rods.

If loading on rod is simple, accurate theoretical analysis is still possible. Skalak and Folk et al.^[1] analyzed cases of impact of two semi-infinite rods and step pulse on a rod respectively by the integral transform technique. The expressions of the solutions include the Airy integral. The Airy integral can predict the oscillation with high frequency in the responses^[2]. Although many higher-order stress effects present in the exact theory of waves in rods, the predominant effect causing deviation of the far-field pulse shape from that predicted by elementary theory is that of lateral inertia.

Whether the solutions obtained from Love theory or from exact theory, these must be solved by numerical technique. The solutions are only given for a few simple loading cases. For practical applications, the shape of impact load is approximated to half a sinusoid. However, it is very difficult to obtain explicit solutions for such a case, the numerical simulations by finite element method are helpful for us to understand the responses of rods to impact loads.

From the dispersion curves shown in Fig. 1, if waves propagating in rods are dominated by the first longitudinal mode, the harmonic waves with longer wavelength (i.e. lower frequency) will propagate faster than the harmonic waves with shorter wavelength (i.e. higher frequency). The longer the traveling distance along axis, the bigger the time lag between these waves. Based on Fourier Theory, the patterns of transient waves will distort during propagating, the oscillation in the waveforms takes place, and the center frequency of the oscillation reduces with increase of the traveling distance.

Because of influences of higher modes, different particles in a cross-section will have different velocities in the axial direction. The results computed by the finite element program ANSYS/LS-DYNA are shown in Fig. 2 for the particles at the position of 0.5R from the axis of rod in three different sections with distances of 0, 0.5D and 1D from the top (to avoid the phase shift caused by low pass filtering, the time corresponding to start of particle velocity responses is delayed). The parameters for calculation are D=0.2m, L/D=10, ρ =2500kg/m³, E=40Gpa, v =0.2 (same as below), where D is the diameter of rod, L/D is the ratio of the length to the diameter of rod, ρ is the density of rod material, E is modulus, v is Poisson's ratio.

The oscillation caused by dispersion of waves can be seen in Fig. 2. The traveling distance of reflection from the free end of rod is two times the length of rod. This can be regarded as the case of traveling distance increasing. It is concluded that when the traveling distance increases, the duration of the first pulse increases and the center frequency of narrow-band oscillation decreases.



Figure 2 The axial particle velocities in different cross-sections

2.3 Analysis on the response of cross-sections

The displacements in z direction (i.e. axial direction) can be expanded in terms of Jacobi polynomials in radial coordinate ^[1]:

$$u_{z} = \sum_{n=0}^{\infty} w_{n}(\alpha) u_{n}(z,t)$$
(3)

where,

 $\alpha = r_{R}', w_{n}(\alpha) = 1 + \sum_{k=1}^{n} (-1)^{k} \left(\frac{n}{k}\right) \frac{(n+1)_{k}}{k!} \alpha^{2k}, \\ \left(\frac{n}{k}\right) = \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!}, \\ \left(\beta\right)_{k} = \beta(\beta+1)(\beta+2)\cdots(\beta+k-1).$

When n=0,1,2, $w_0(\alpha) = 1$, $w_1(\alpha) = 1 - 2\alpha^2$, $w_2(\alpha) = 1 - 6\alpha^2 + 6\alpha^4$. w_0, w_1, w_2 are corresponding to the first three longitudinal modes respectively. The variations of $w_1(\alpha)$ and $w_2(\alpha)$ with r are shown in Fig. 3. The zeros of $w_1(\alpha) = 1 - 2\alpha^2$ are $\alpha = \pm \sqrt{2/2} = \pm 0.707$, they are corresponding to extreme points of $w_2(\alpha) = 1 - 6\alpha^2 + 6\alpha^4$. The zeros of $w_2(\alpha) = 1 - 6\alpha^2 + 6\alpha^4$.



Fig.3 The radial variation in displacement direction for 2nd and 3rd longitudinal modes

In general, the particle velocity responses of cross-sections are dominated by the 1st mode. When high frequency contents of waves increase, the 2nd mode of waves begins to affect the particle velocity responses, the position for minimum oscillation is 0.707R from the axis. If the 3rd mode of waves takes part, the position for minimum oscillation will change correspondingly. The position for minimum oscillation can be calculated as follows:

$$(1 - 2\alpha^{2})u_{1}(z, t) + (1 - 6\alpha^{2} + 6\alpha^{4})u_{2}(z, t) = 0$$
$$\alpha^{2} = \frac{1}{2} + \frac{u_{1}}{(6u_{2})} \pm \frac{u_{1}}{(6u_{2})} \sqrt{1 + 3\left(\frac{u_{2}}{u_{1}}\right)^{2}}$$

where $\alpha^2 = \frac{1}{2} + \frac{u_1}{(6u_2)} - \frac{u_1}{(6u_2)} \sqrt{1 + 3\left(\frac{u_2}{u_1}\right)^2}$ is in range of 0 to R, $\alpha^2 = \frac{1}{2} + \frac{u_1}{(6u_2)} + \frac{u_1}{(6u_2)} \sqrt{1 + 3\left(\frac{u_2}{u_1}\right)^2}$ is meaningless. When $u_2 << u_1$, $\sqrt{1 + 3\left(\frac{u_2}{u_1}\right)^2}$ can be expanded in terms of Taylor series. If the first two terms are kept, α^2 is given by $\alpha^2 = \frac{1}{2} - \frac{1}{4} \left(\frac{u_2}{u_1}\right)$

The more the high frequency contents of the impact pulse are, the more the modes of waves are. The Poisson's ratio can change the proportion of different mode influence. These factors will affect the responses of particles in the top and the position for minimum oscillation. When the pulse width is bigger than R, the positions are at about 0.7R, 0.6R, 0.5R from the axis for v = 0,0.2,0.45 respectively.

The center frequency of narrow-band oscillation is near in C_R/2R~ C_s/2R as proposed by Cheng^[3], or is estimated from f₀=C₀ /(β .R), which is derived from the results of finite element simulation, where C_R is the velocity of Rayleigh waves, C_s is the velocity of shear waves, $\beta \approx 3.8$ - v.

With the increase of propagating distance of waves from the top, the influence of higher modes reduce, the differences between the responses of the particles in one cross-section reduce also.

3. Influences of the loading area and the impulse width on responses

3.1 Loading uniformly distributed on the top of rods

According to the classical 1-D rod theory, the responses of particles in one cross-section are the same. The first pulse of the responses on the top is corresponding to the incident wave, and on the free top, the amplitude of waves reflected from the free end of the rod is two times that of the incident waves. However, influenced by dispersion of longitudinal modes, each particle in one cross-section has different response to impact loading. The increase of propagation distance will reduce influence of higher modes and the responses in one cross-section approach each other. The responses between different cross-sections are still different because of dispersion of the fundamental longitudinal mode.

The waveform and amplitude of waves change during waves propagating. Thus, the classical rod theory is not available to analyze the stress wave propagating in a rod. This phenomenon can be confirmed by the simulation results shown in Fig. 4(a). The velocity responses on the top are different for two positions with different distances from the center of the rod. In this simulation case, the impulse width is D, the ratio of L/D is 10, where D is 0.5m, L is the length of the rod.

The larger the impulse width is, the smaller the influence of dispersion is. The waveform and its magnitude will not change during wave propagation. When the impulse width is larger than or equal to 4D, the fundamental longitudinal mode becomes dominant and each particle on the top of rod has similar response and the waves propagate with speed of C_0 . As shown in Fig. 4(b), for the responses on the top of the rod, the amplitude of the free-end reflected waves is two times that of the first pulse. The responses on the top of the rod can be analyzed by one-dimensional approximation.

Since the above analysis of the dispersion is based on the assumption of rods with uniform cross-section, the rods must have uniform cross-section in a range of at least one impulse width below the top of rods to meet the requirement of one-dimensional approximation. If the impedance of rods varies in this range, the wavefront of reflections from the location of impedance variation are non-plane, only the waves reflected from locations below the top greater than 4D can be analyzed approximately by the 1-D rod theory. The reflecting locations are also very important for 1-D approximation analysis.



(a) impulse width equal to D; and (b) impulse width equal to 4D

3.2 Loading distributed on part of the top of rods

In integrity testing of piles, loading is distributed on small circular area around the center of piles. The discussion on influence of size of the loading area to the propagation of stress waves is very useful in practical testing.

When loading is distributed on part of the top, besides the longitudinal modes of wave, other type of waves will affect the responses of particles, such as Rayleigh waves along the top. The first pulse of the responses on the top is affected not only by the longitudinal mode of waves but also by Rayleigh waves, while responses to reflections from the rod body are mainly affected by the longitudinal mode of waves.

The geometric attenuation of Rayleigh waves with the radial traveling distance r is $r^{-1/2}$, if non-reflective boundaries are applied to the surface of rods, the Rayleigh wave reflection from free boundary will not be taken into consideration. The axial velocities of the particles with longer distance from loading area are smaller, as shown in Fig. 5(a), but with influences of the longitudinal modes of

waves, the axial velocities do not attenuate with the traveling distance, and the narrow-band oscillation occurs, as in shown in Fig. 5(b). The zeros of the oscillation are nearly same for different positions on the top. The center frequency of narrow band oscillation and the position for minimum oscillation are similar to the case of loading on the whole top of the rod.



Figure 5 The axial velocity responses to (a) Rayleigh waves and; (b) several types of waves

The increase of the impulse width can reduce dispersion of waves and influence of higher modes of waves. Relative to influence of the higher modes, the three-dimensional wave effect caused by loading distributed on part of the rod is strong. One dimensional wave approximation cannot be achieved by increasing the impulse width.

For a case of a/R=1/8, D=0.5m, L/D=6, impact pulse width of 4D, where a is radius of loading area, the velocity responses are shown in Fig. 6 for the positions of 9/10R and 3/5R from the center of axis. From the shapes and amplitudes of impact pulse and the pulse of reflection from the free end in the responses, propagation of waves does not comply to 1-D wave theory. It is noted that the position for minimum oscillation is at 0.6R from the center as shown in Fig. 6(a)

For a case of a/R = 3/8, D=0.5m, L/D=10, impact pulse width of 4D, the velocity responses are shown in Fig. 6(b) for particles on the top and in the cross-section D below the top. It can be seen that the increase of loading area can improve the shapes and amplitudes of the first pulse and the reflection pulse.

When the impact pulse width is long enough, the depth in which waves propagate in three dimensions reduces as the loading area increases. For a case of a/R < 1/4, although waves propagate approximately as plan waves after the ratio of traveling distance to D reaches a value (the traveling distance at least is greater than the effect depth of three dimensional waves, 4D), the first pulse of the responses does not approach one of the plane waves, and shall not be regarded as one of incident waves. When a/R > 1/4 and the impact pulse width is larger than 4D, the pulse of the plane wave can be approximated by the first pulse of the responses.



4. Influences of 3D wave propagation in the integrity testing of piles

Piles are buried structures, the impact-echo technique has been widely used in integrity testing of piles. The impedance variation of piles can be analyzed from the reflected wave signals measured on the top based on 1-D wave theory. The higher the frequencies of reflections are, the more information contained about the impedance variation. It is very important to analyze the impedance variation. Increase of impact pulse width makes the effects of waves unobvious, and losses of useful information on impedance variation. This is especially disadvantage for integrity testing on the shallow parts. The analysis of impedance variation from the measured reflection signals is an inverse procedure, the useful reflections with higher frequencies are beneficial to the uniqueness of results. However, in actual integrity testing, the waves with lower frequencies are necessary for one-dimensional approximate analysis of signals. Propagation of the dispersive and non-plane waves in piles has great influences on integrity testing of piles.

These influences are mainly as follows:

(1) The size of loading area relative to the top area will affect wave propagation. The smaller the size

is, the stronger the non-plane of waves is. If pile impedance varies in the influence range of non-plane waves, the responses to the reflections are different for the different particles on the top of piles not only in amplitudes but also in patterns. Because the pile body cannot be regarded as 1-D rod any more, the location of impedance variation in radial direction will affect the reflections. The influence of 3D waves on integrity testing of piles can be examined through the numerical simulation.

The model parameters for calculation are as follows: impact force pulse width is 2D, the range of bulge is from 2D to 2.4D below the top, the radius of the bulge part increases from R to 7/5R in circumferential direction 0°-180°, D=0.5m, L/D=10, v=0.2, a/R =1/4. The results are shown in Fig. 7. The responses are different for the positions of 0.88R and 0.65R from the center of axis, even at the circle of 0.65R, the velocity response of the particle above the location of impedance variation (legend I in Fig. 7) is different from others. Because of the effect of 3D waves, analysis of the position and degree of impedance variation is difficult.

- (2) Comparison between the amplitudes and phases of the reflected and incident wave pulses of responses is used to analyze the impedance variation. However, the first pulse of responses isn't corresponding to incident wave.
- (3) The degree of oscillation increases when impact pulse width decreases. The reflections over-lapped with the oscillation are not easily distinguished from the oscillation.



Fig.7 The velocity responses at positions of (a) 0.88R; and (b) 0.65R from the axis

5. Integrity testing on the shallow part of piles

For integrity testing of the parts with depth greater than 4D, the waves induced by impacting propagate as non-dispersively plane waves by increasing loading area and impact pulse width. However, for integrity testing on the shallow parts, propagation of waves is very complex.

When the ratio of the length to the diameter is bigger than 10, as mentioned above, waves reflected from the pile toe are approximate to non-dispersively plane waves. The waves propagate upwards through the part with impedance variation, and propagate downwards after reflected at the top again. The waves will be reflected upwards again when the downward waves encounter the impedance variation. Although the multiple reflections from the impedance variation interfere with the reflection from the toe and its sequential reflections more or less, such interference can be minimized using appropriate low-pass filtering. If the waves from the toe reflected at the top are taken as the incident waves, the waves and their sequential waves reflected from the part with impedance variation can be analyzed approximately based on 1-D theory.

For a model pile with a neck, some parameters are as follows: the range of neck is from 2D to 2.5D below the top, the radius is reduced from R to 0.5R in circumferential direction 0°-180°, D=0.2m, L/D=10, impact force pulse width is 4D, Poisson's ratio is 0.2, a/R = 1/5. The numerical results are shown in Fig. 8.



Fig.8 The particle velocity responses to reflections from the impedance variation and free end

It can be seen that the particle velocity responses at three different positions on the top to reflection from the impedance variation are different. It is very difficult to distinguish the reflection from the response. However, the particle velocity responses to reflection from the toe and its sequential reflection from the impedance variation are almost the same at different positions on the top, and can be analyzed by classical rod wave theory.

When the location of impedance variation is near to the top, the reflection from the toe and the sequential reflection will overlap each other. Such reflections from the impedance variation are difficult to distinguish.

In general, it is better to analyze the particle velocity response at the position with minimum oscillation (0.5R-0.7R for different Poisson's ratios and impulse width). However, the position for minimum oscillation, which is obtained from analysis of waves propagating in rods with uniform cross-section, is not available for rods with impedance variation in shallow part. The optimum position for response is influenced by location of impedance variation.

6. Conclusions

Due to the influence of dispersion of the longitudinal modes, wave propagation in rods cannot be analyzed based on classical rod wave theory. There is narrow-band oscillation in the axial particle velocity response on the top of rod. The amplitude and center frequency of oscillation is different for different position. The oscillation is caused by the higher longitudinal modes. The position for minimum of oscillation ranges from 0.5R to 0.7R, and is influenced by Poisson's ratio and impulse width. When impact pulse width λ is greater than 4D and the reflection from the location greater than 4D below the top, the response on the top can be analyzed approximately based on 1-D wave theory.

When loading is distributed on part area of the top of rod, Rayleigh waves will have a great influence on the first pulse of the velocity response. Increase of the ratio of the loading area to the area of the top can reduce the depth in which waves propagate in three dimensions, and the Rayleigh wave components of the first pulse. When a/R>1/4 and the width is greater than 4D, the waves reflected from the locations greater than 4D below the top will approach plan waves, and the pulse of the incident waves will approach the first pulse of the response.

For integrity testing of shallow part of piles, because of the effect of three dimension waves near the top, the velocity responses to reflection are different for different particles on the top, and overlapped by oscillation with higher frequencies. The response with minimum oscillation is beneficial to analyze the reflection. The reliability of analysis can be improved by analysis of reflection from the toe and its sequential reflection.

Confined by the loading area and impact pulse width, one dimensional approximation theory is not available for piles with large diameters.

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